

Adding and Subtracting Rational Numbers Using Number Lines

1 GETTING THE IDEA

The **integers** are the set of whole numbers, their opposites, and zero.

..., -2, -1, 0, 1, 2, ...

A **rational number** is a number that can be expressed as a ratio of two integers $\frac{a}{b}$, where $b \neq 0$.

Rational numbers include fractions, terminating decimals, and repeating decimals. Examples of rational numbers are -3.8 , $-\frac{1}{2}$, 0 , $1.\overline{7}$, and $12\frac{3}{8}$.

The **additive inverse** of a number is its opposite. A number and its additive inverse have the same absolute value. This means they are each the same distance from 0 on the number line.

Example 1

Find the sum: $-3 + 3$

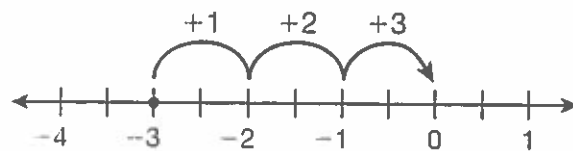
Strategy Use a number line.

Step 1 Locate -3 on the number line.

Count 3 units to the right of -3 . Plot a point there for -3 .

Step 2 Add 3.

Move 3 units to the right of -3 . The arrow points to 0, which is the sum.



Solution $-3 + 3 = 0$

Example 2

The temperature of a solid is -13.2°C . The solid will melt when its temperature is raised 4.5°C . At what temperature will the solid melt?

Strategy Use a number line.

Step 1 Write an expression to represent the problem.

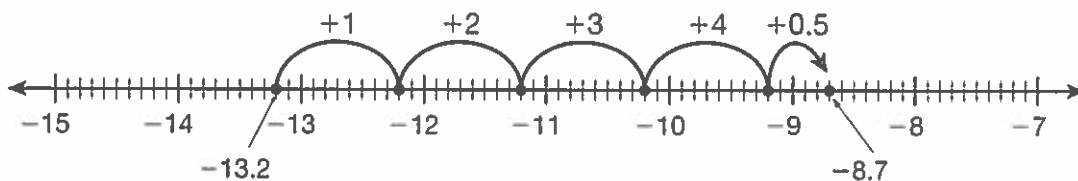
Since the temperature is raised, write an addition sentence.

$$-13.2 + 4.5$$

Step 2 Use a number line to add.

Locate -13.2 on the number line.

From -13.2 , move 4 units to the right. Then move 0.5 more.



Solution The solid melts at -8.7°C .

You do not need to use a number line to add rational numbers. You can use rules instead.

- To add rational numbers with the same sign, add the absolute values of the addends. Then give the sum the same sign as the addends.
- To add rational numbers with different signs, subtract the lesser absolute value from the greater absolute value. Give the sum the sign of the addend with the greater absolute value.
- To subtract a rational number, add its opposite.

Example 3

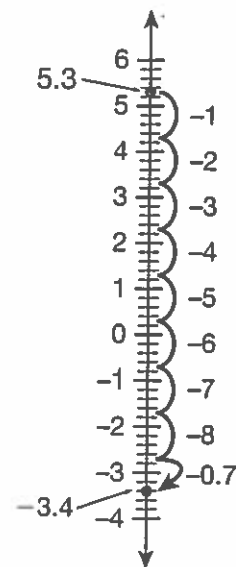
A stock price rose 5.3 points in one week. The stock price fell 8.7 points the next week. How did the stock price change during the two-week period?

Strategy Use a vertical number line to add rational numbers.

Step 1 Write an expression to represent the problem.
Add to find the change in the stock prices.
 $5.3 + (-8.7)$

Step 2 Use a vertical number line to add a negative number.
Locate 5.3 on the vertical number line.
From 5.3, move 8 units down. Then move 0.7 unit down.
 $5.3 + (-8.7) = -3.4$

Solution During the two weeks, the stock price fell 3.4 points.



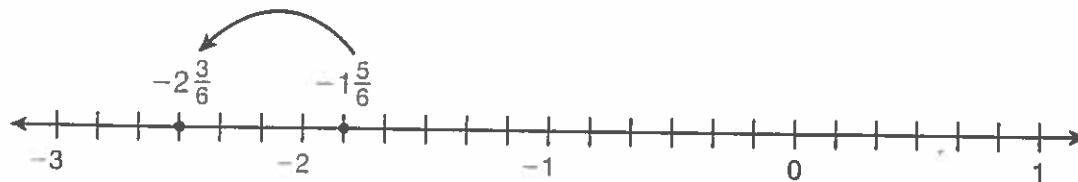
Example 4

Subtract: $-1\frac{5}{6} - \frac{2}{3}$

Strategy Use a number line.

Step 1 Write $\frac{2}{3}$ with a denominator of 6.
 $\frac{2}{3} = \frac{4}{6}$
 $-1\frac{5}{6} - \frac{2}{3} = -1\frac{5}{6} - \frac{4}{6}$

Step 2 Use a number line to subtract.
Locate $-1\frac{5}{6}$ on the number line.
Move $\frac{4}{6}$ units to the left of $-1\frac{5}{6}$ to subtract.



To subtract also means to add its opposite.

$$-1\frac{5}{6} + \left(-\frac{4}{6}\right) = -2\frac{3}{6} = -2\frac{1}{2}$$

Solution $-1\frac{5}{6} - \frac{2}{3} = -2\frac{1}{2}$

2 COACHED EXAMPLE

A scuba diver uses negative numbers to represent her depth below sea level. She started at a depth of -1.5 meters. Then she dove down to a depth of -4.2 meters. What is the difference between those depths?

Write an expression to represent the problem.

To find the difference, the operation to use is _____.

$$-1.5 \square (-4.2)$$

To subtract means to _____ its opposite.

$$-1.5 \square (-4.2) = -1.5 \square (4.2)$$

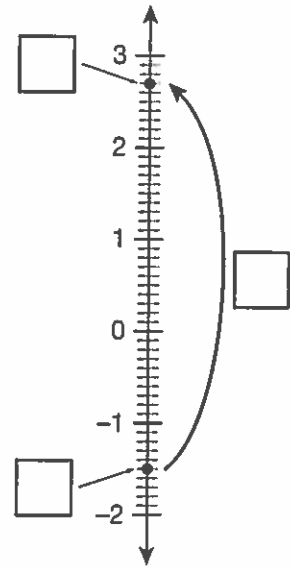
Use a vertical number line to _____.

Locate _____ on the number line.

From _____, move _____ units _____.

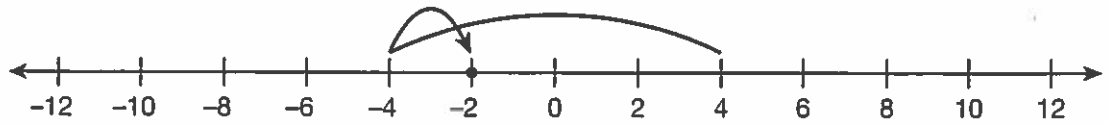
$$-1.5 \square (-4.2) = \underline{\hspace{2cm}}$$

The difference is _____ meters. This is the total distance the diver dove to reach her new depth.

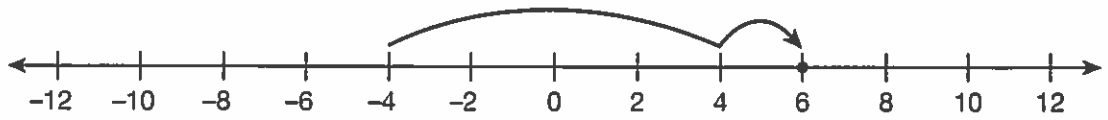


1 Which number line represents $-4 - (-8) + 2$?

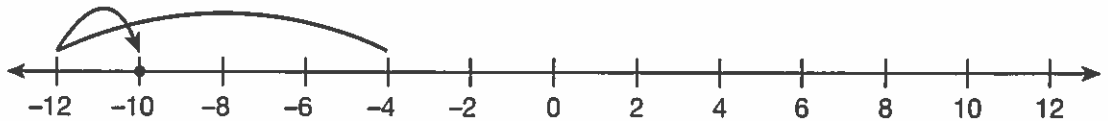
A.



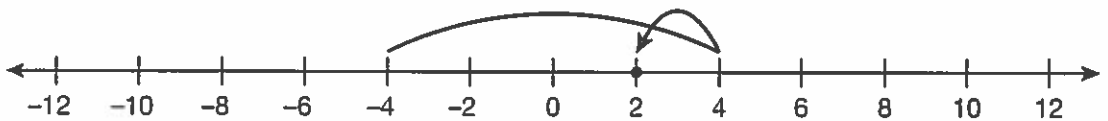
B.



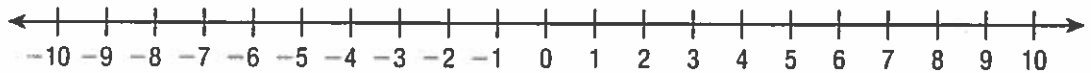
C.



D.



Use the number line to answer questions 2–5.



2 What is $2 - (-5)$?

- A. -7
- B. -3
- C. 3
- D. 7

3 What is $-5 + 2$?

- A. -7
- B. -3
- C. 3
- D. 7

4 What is $-5 + (-2)$?

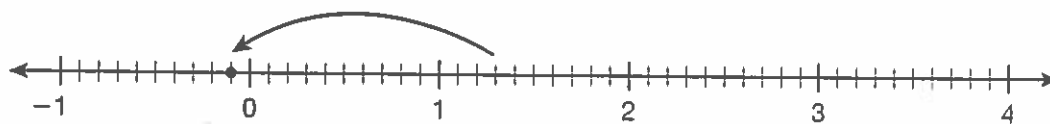
- A. -7
- B. -3
- C. 3
- D. 7

5 What is $-2 - (-5)$?

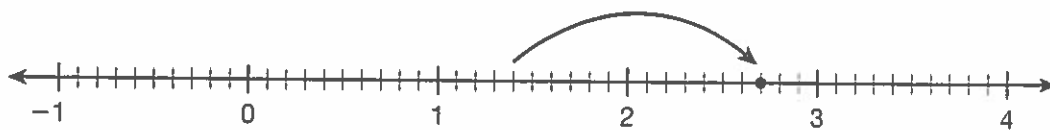
- A. -7
- B. -3
- C. 3
- D. 7

6 Which number line represents $1.3 - 1.4$?

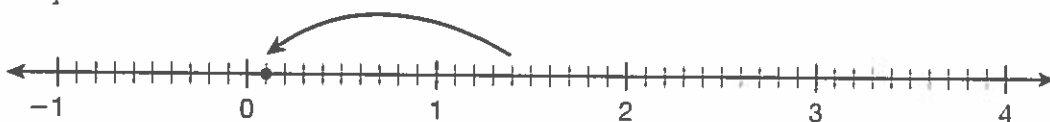
A.



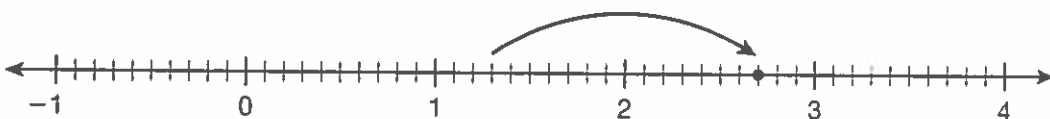
B.



C.



D.



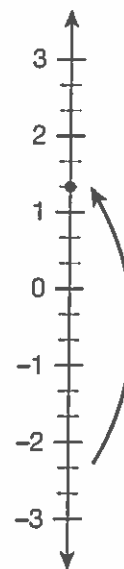
7 Which equation is represented by the number line at the right?

A. $1\frac{1}{3} - 2\frac{1}{3} = -1$

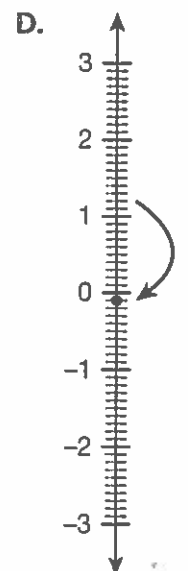
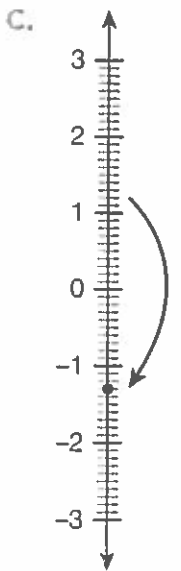
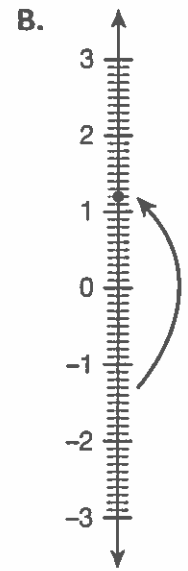
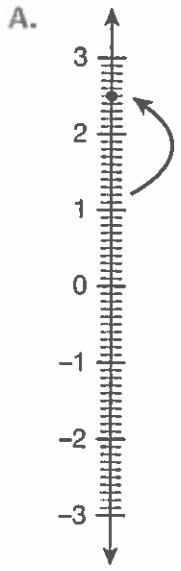
B. $-2\frac{1}{3} + 1\frac{1}{3} = -1$

C. $-2\frac{1}{3} - (-3\frac{2}{3}) = 1\frac{1}{3}$

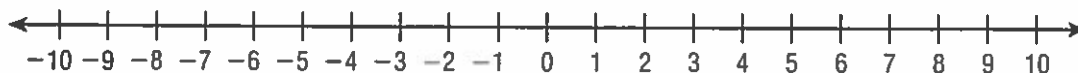
D. $-3\frac{2}{3} + 2\frac{1}{3} = -1\frac{1}{3}$



- 8 The temperature at 2 A.M. is -1.3°C . The temperature at 10 A.M. is 1.2°C . Which vertical number line represents the change in temperature from 2 A.M. to 10 A.M.?



Use the number line to answer questions 9 and 10.



9 What is $-5 - (-2) - 3$?

- A. -10
- B. -6
- C. -4
- D. 0

10 What is $3 + (-4) - (-2)$?

- A. 9
- B. 4
- C. 1
- D. -1

11 A submarine at a depth of $-25\frac{2}{3}$ feet descended $5\frac{1}{2}$ feet. The submarine then ascended $2\frac{1}{6}$ feet.

Part A

Write an expression to determine the depth of the submarine.

Part B

What is the depth of the submarine? Show how you found the depth using a number line.

A horizontal number line with arrows at both ends. There is a tick mark at 0 on the left side.